

Question 4

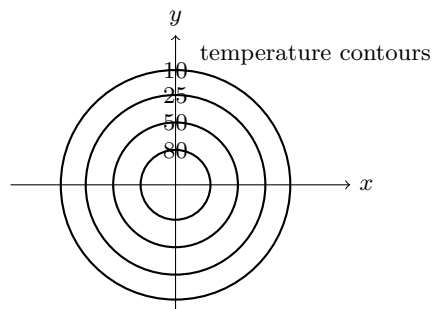
A thin plate has temperature

$$T(x, y) = 100e^{-(x^2+4y^2)/8} \quad (^\circ\text{C}).$$

Find the 50° isotherm, its semiaxes and enclosed area. Determine the warmest point and explain the contour spacing as temperature decreases.

Question 4 – Solution

Strategy Set T equal to a constant and take logarithms.



Isotherm Setting $T = 50$ gives $e^{-(x^2+4y^2)/8} = 1/2$, hence

$$x^2 + 4y^2 = 8 \ln 2.$$

The semiaxes are $\sqrt{8 \ln 2}$ and $\sqrt{2 \ln 2}$, so the area is $\boxed{4\pi \ln 2}$.

Maximum and spacing The maximum is 100° at $(0,0)$. For level $0 < c < 100$, $x^2 + 4y^2 = 8 \ln(100/c)$; lower temperatures correspond to larger ellipses, with nonlinear spacing caused by exponential decay.