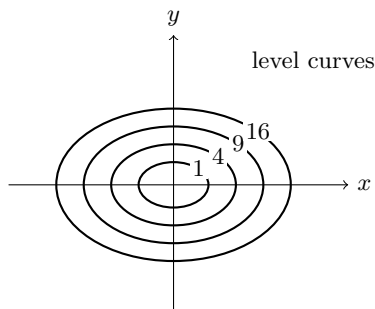


**Question 2**

For  $f(x, y) = 9 - 4(x - 1)^2 - (y + 2)^2$ , find the range, global extrema, and level curves  $f = c$ . Describe how the curves change as  $c$  varies.

## Question 2 – Solution

**Strategy** The subtracted squares are nonnegative; solve  $f = c$  for the level sets.



**Range and extrema** The maximum is  $\boxed{9}$  at  $(1, -2)$ . There is no minimum because  $f \rightarrow -\infty$  as  $\|(x, y)\| \rightarrow \infty$ , so the range is  $\boxed{(-\infty, 9]}$ .

**Level curves** For  $c < 9$ ,

$$4(x - 1)^2 + (y + 2)^2 = 9 - c,$$

an ellipse centered at  $(1, -2)$  with semiaxes  $\sqrt{9 - c}/2$  and  $\sqrt{9 - c}$ . For  $c = 9$  the level is one point; for  $c > 9$  it is empty.