

Question 1

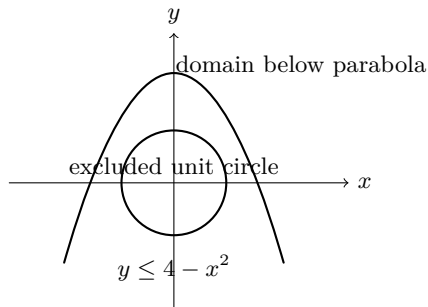
Find and sketch the domain of

$$f(x, y) = \frac{\sqrt{4 - x^2 - y^2}}{\ln(x^2 + y^2)}.$$

State which boundary pieces are included and whether the domain is bounded, open, closed, or neither.

Question 1 – Solution

Strategy Enforce the square-root, logarithm, and denominator restrictions simultaneously.



Restrictions We need $4 - x^2 - y \geq 0$, $x^2 + y^2 > 0$, and $\ln(x^2 + y^2) \neq 0$. Thus

$$D = \{(x, y) : y \leq 4 - x^2, x^2 + y^2 \neq 0, 1\}.$$

Boundary and topology The entire parabola $y = 4 - x^2$ is included; it does not meet the unit circle or the origin. The origin and entire unit circle are excluded. The region is unbounded downward. It is not open because the parabolic boundary is included, and not closed because excluded circle points are limit points.

Check Every remaining point makes the radicand nonnegative and the denominator defined and nonzero.