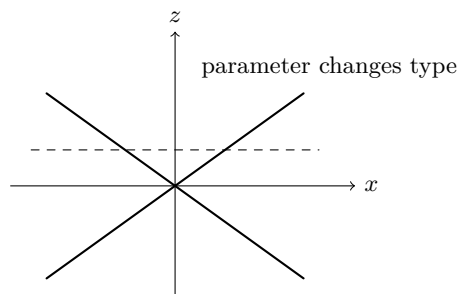


Question 10

Classify the family $x^2 + y^2 - z^2 = \lambda$ for $\lambda > 0$, $\lambda = 0$, and $\lambda < 0$. Describe how the topology changes as λ passes through zero and give the vertices when they exist.

Question 10 – Solution

Strategy Divide by $|\lambda|$ and inspect which side has the single positive term.



$\lambda > 0$ The form $x^2/\lambda + y^2/\lambda - z^2/\lambda = 1$ is a hyperboloid of one sheet, connected, with axis z .

$\lambda = 0$ The equation $x^2 + y^2 = z^2$ is a double circular cone with vertex at the origin.

$\lambda < 0$ Writing $z^2/|\lambda| - x^2/|\lambda| - y^2/|\lambda| = 1$ gives a hyperboloid of two sheets. Its vertices are $\boxed{(0, 0, \pm\sqrt{|\lambda|})}$.

Transition At zero, the one-sheet surface pinches to a cone and separates into two components.