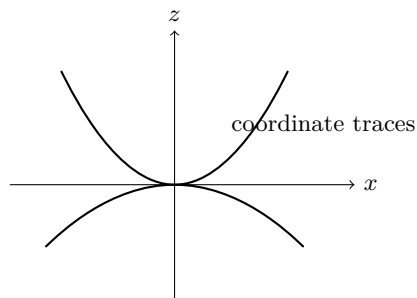


Question 8

A quadric has the traces $z = x^2$ in the plane $y = 0$ and $z = -y^2$ in the plane $x = 0$. Every horizontal trace $z = k \neq 0$ is a hyperbola. Determine a simplest equation for the surface and explain whether the trace information uniquely fixes all coefficients.

Question 8 – Solution

Strategy Combine the two vertical traces into a quadratic having the required restrictions.



Simplest model The equation

$$z = x^2 - y^2$$

has exactly the stated coordinate-plane traces and horizontal hyperbolas. It is a hyperbolic paraboloid.

Uniqueness issue The given coordinate traces force the coefficients of x^2 and y^2 in a diagonal model, but without an assumption excluding a mixed term, $z = x^2 - y^2 + cxy$ has the same traces in $x = 0$ and $y = 0$. Its horizontal traces remain hyperbolic when the quadratic form is indefinite.

Conclusion The simplest axis-aligned equation is determined, but the data do not uniquely exclude rotated saddles.