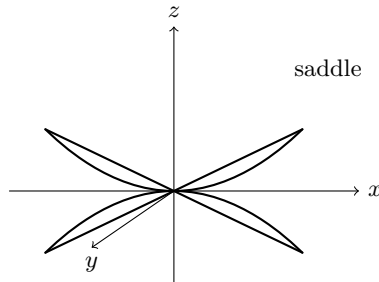


Question 6

Analyze the hyperbolic paraboloid $z = x^2 - 4y^2$. Find the traces in $x = 0$, $y = 0$, $z = 0$, and $z = k \neq 0$. Explain how these traces establish the saddle shape.

Question 6 – Solution

Strategy Substitute each fixed coordinate and classify the resulting planar curve.



Vertical traces In $y = 0$, $z = x^2$ opens upward. In $x = 0$, $z = -4y^2$ opens downward. Their opposite concavities create the saddle.

Horizontal traces At $z = 0$, $x^2 - 4y^2 = 0$, so $x = \pm 2y$, two intersecting lines. For $k > 0$, $x^2 - 4y^2 = k$ is a hyperbola opening in the x -direction; for $k < 0$, it opens in the y -direction.

Conclusion The origin is neither a local maximum nor minimum because nearby traces rise in one direction and fall in the perpendicular direction.