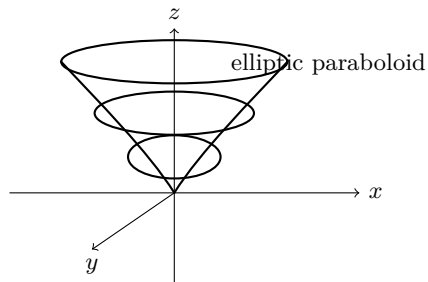


Question 5

The surface $z = x^2 + 4y^2 - 2x + 8y + 7$ is cut by the plane $z = 10$. Identify the surface, locate its vertex, and find the semiaxes and area of the resulting planar region.

Question 5 – Solution

Strategy Complete squares in x, y ; a horizontal cut of an elliptic paraboloid is an ellipse.



Standard form We obtain

$$z = (x - 1)^2 + 4(y + 1)^2 + 2.$$

Thus the surface is an elliptic paraboloid opening upward with vertex $(1, -1, 2)$.

Cut at $z = 10$ The boundary satisfies $(x - 1)^2 + 4(y + 1)^2 = 8$, or

$$\frac{(x - 1)^2}{8} + \frac{(y + 1)^2}{2} = 1.$$

Its semiaxes are $2\sqrt{2}$ and $\sqrt{2}$, so the enclosed area is $\pi(2\sqrt{2})(\sqrt{2}) = 4\pi$.