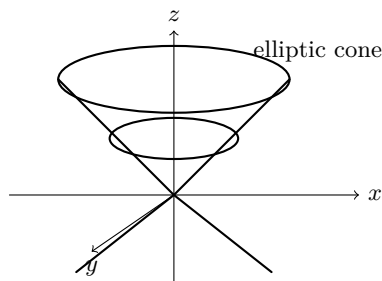


Question 4

Classify the surface $4x^2 + y^2 - 9z^2 = 0$. Describe its traces in the planes $z = k$, $x = k$, and $y = k$, and find the lines obtained in the trace $y = 0$.

Question 4 – Solution

Strategy A homogeneous quadratic with two positive squares and one negative square describes a cone.



Classification The equation is $4x^2 + y^2 = 9z^2$, an elliptic cone with vertex at the origin and axis the z -axis.

Traces For $z = k \neq 0$, $4x^2 + y^2 = 9k^2$ is an ellipse; for $z = 0$ it is only the vertex. For fixed $x = k$ or $y = k$, the traces are hyperbolas when the fixed value is nonzero and intersecting lines when it is zero.

$y = 0$ Then $4x^2 = 9z^2$, so $z = \pm \frac{2}{3}x$.