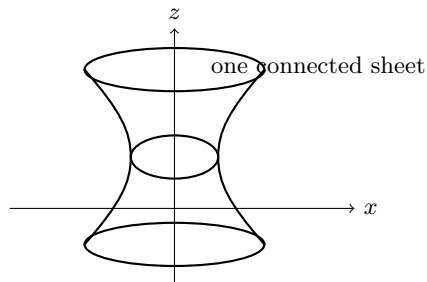


Question 2

Analyze $x^2 + 4y^2 - z^2 - 2x + 16y + 4z - 12 = 0$. Put it in standard form, classify it, identify its axis, and describe the traces $z = 2$ and $z = 2 + k$.

Question 2 – Solution

Strategy Complete squares and inspect the signs. Then substitute fixed z -levels.



Standard form Simplification gives

$$(x-1)^2 + 4(y+2)^2 - (z-2)^2 = 25, \quad \boxed{\frac{(x-1)^2}{25} + \frac{(y+2)^2}{25/4} - \frac{(z-2)^2}{25} = 1}.$$

It is a hyperboloid of one sheet centered at $(1, -2, 2)$ with axis parallel to z .

Traces At $z = 2$ the waist ellipse is $(x-1)^2 + 4(y+2)^2 = 25$. At $z = 2+k$ it becomes $(x-1)^2 + 4(y+2)^2 = 25 + k^2$, so ellipses expand as $|k|$ grows.