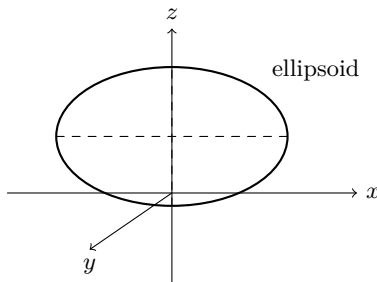


Question 1

Put $4x^2 + y^2 + 9z^2 - 8x + 6y - 36z + 4 = 0$ in standard form. Classify the surface and give its center, semiaxes, and coordinate-direction extreme points.

Question 1 – Solution

Strategy Complete the square separately in x, y, z , then divide by the resulting constant.



Standard form Completing squares gives

$$4(x - 1)^2 + (y + 3)^2 + 9(z - 2)^2 = 45,$$

so

$$\frac{(x - 1)^2}{45/4} + \frac{(y + 3)^2}{45} + \frac{(z - 2)^2}{5} = 1.$$

Geometry This is an ellipsoid centered at $(1, -3, 2)$ with semiaxes $3\sqrt{5}/2$, $3\sqrt{5}$, and $\sqrt{5}$. Its axial extreme points are

$$(1 \pm 3\sqrt{5}/2, -3, 2), \quad (1, -3 \pm 3\sqrt{5}, 2), \quad (1, -3, 2 \pm \sqrt{5}).$$

Verification Substitution of any listed endpoint makes exactly one normalized square equal to 1.