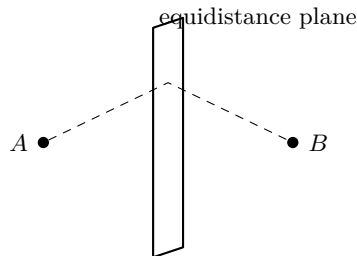


Question 10

Prove that the set of points equidistant from two distinct points A and B is a plane perpendicular to $\overline{AB}$ through its midpoint. Then find that plane for $A = (1, -2, 4)$ and $B = (5, 0, -2)$.

Question 10 – Solution

Strategy Expand $\|\vec{r} - \vec{a}\|^2 = \|\vec{r} - \vec{b}\|^2$ and cancel $\vec{r} \cdot \vec{r}$.



General proof Expansion gives

$$2(\vec{b} - \vec{a}) \cdot \vec{r} = \|\vec{b}\|^2 - \|\vec{a}\|^2.$$

This is a plane with normal $\vec{b} - \vec{a}$. Substituting $(\vec{a} + \vec{b})/2$ verifies that the midpoint lies on it.

Application Here $\vec{b} - \vec{a} = \langle 4, 2, -6 \rangle$ and the midpoint is $(3, -1, 1)$. Thus

$$4(x - 3) + 2(y + 1) - 6(z - 1) = 0,$$

or $\boxed{2x + y - 3z = 2}$.

Verification Direct substitution of the midpoint gives $6 - 1 - 3 = 2$.