

Question 7

Find the plane in the family containing the intersection line of $x + y + z = 2$ and $2x - y + 3z = 1$ that is perpendicular to the plane $x - 2y + z = 4$.

Question 7 – Solution

Strategy Apart from $2x - y + 3z = 1$ itself, every plane through the common line has equation $(x + y + z - 2) + \lambda(2x - y + 3z - 1) = 0$. The omitted plane has normal dot product $7 \neq 0$ with the given normal, so it cannot qualify. Choose λ so the family normal is perpendicular to the given normal.

pencil of planes



Normal condition The family normal is $\langle 1 + 2\lambda, 1 - \lambda, 1 + 3\lambda \rangle$. Dotting with $\langle 1, -2, 1 \rangle$ gives 7λ , so $\lambda = 0$.

Result The required plane is therefore simply

$$\boxed{x + y + z = 2}.$$

Verification Its normal $\langle 1, 1, 1 \rangle$ has dot product $1 - 2 + 1 = 0$ with $\langle 1, -2, 1 \rangle$, and it contains the stated intersection line.