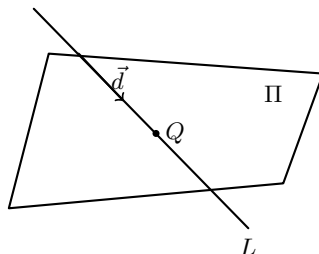


Question 6

Classify the relationship between the line $L : \vec{r} = \langle 1, 2, 0 \rangle + t \langle 2, -1, 3 \rangle$ and the plane $\Pi : 3x + y - z = 5$. If they intersect, find the point and angle at which the line meets the plane.

Question 6 – Solution

Strategy Substitute the line into the plane. The line-plane angle is complementary to the angle between the direction and the normal.



Intersection Substitution gives $3(1 + 2t) + (2 - t) - 3t = 5$, so $2t = 0$ and $t = 0$. Thus $Q = \boxed{(1, 2, 0)}$.

Angle With $\vec{d} = \langle 2, -1, 3 \rangle$ and $\vec{n} = \langle 3, 1, -1 \rangle$,

$$\sin \alpha = \frac{|\vec{d} \cdot \vec{n}|}{\|\vec{d}\| \|\vec{n}\|} = \frac{2}{\sqrt{154}}.$$

Hence $\alpha = \sin^{-1}(2/\sqrt{154}) \approx 9.27^\circ$.

Verification Since $\vec{d} \cdot \vec{n} \neq 0$, the line is not parallel to the plane.