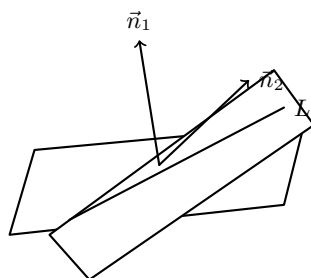


Question 3

Find the line of intersection of the planes $\Pi_1 : x + 2y - z = 3$ and $\Pi_2 : 2x - y + 2z = 1$. Also find the acute dihedral angle between the planes.

Question 3 – Solution

Strategy The intersection direction is perpendicular to both normals. Obtain one point by fixing a convenient coordinate.



Intersection line With $\vec{n}_1 = \langle 1, 2, -1 \rangle$ and $\vec{n}_2 = \langle 2, -1, 2 \rangle$,

$$\vec{n}_1 \times \vec{n}_2 = \langle 3, -4, -5 \rangle.$$

Setting $z = 0$ gives $x + 2y = 3$, $2x - y = 1$, hence $(x, y) = (1, 1)$. Therefore

$$\vec{r} = \langle 1, 1, 0 \rangle + t \langle 3, -4, -5 \rangle.$$

Angle Since $\vec{n}_1 \cdot \vec{n}_2 = -2$, $\|\vec{n}_1\| = \sqrt{6}$, and $\|\vec{n}_2\| = 3$,

$$\theta = \cos^{-1} \left(\frac{2}{3\sqrt{6}} \right) \approx 74.2^\circ.$$