

Question 10

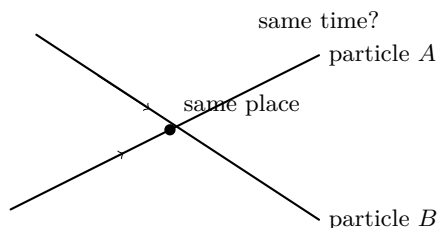
Two particles move according to

$$\vec{r}_A(t) = \langle 1, 0, 2 \rangle + t \langle 2, 1, -1 \rangle, \quad \vec{r}_B(t) = \langle 5, 2, 0 \rangle + t \langle -1, 0, 1 \rangle, \quad t \geq 0.$$

Do their paths intersect? Do the particles collide? Find their minimum separation for $t \geq 0$.

Question 10 – Solution

Strategy Path intersection permits different parameters; collision requires the same time. Minimum separation comes from minimizing the squared relative distance.



Paths Solving $\vec{r}_A(s) = \vec{r}_B(u)$ gives $s = 2, u = 0$, so the paths intersect at $(5, 2, 0)$. The particles do not collide there because they reach it at different times. Directly equating positions at the same t is inconsistent.

Minimum separation The relative position is

$$\vec{r}_A(t) - \vec{r}_B(t) = \langle -4 + 3t, -2 + t, 2 - 2t \rangle.$$

Thus $D^2(t) = 14t^2 - 36t + 24$, minimized at $t = 36/28 = 9/7$. The minimum squared distance is $6/7$, so

$$D_{\min} = \sqrt{6/7} = \frac{\sqrt{42}}{7}.$$

Verification The minimizing time is nonnegative, and the quadratic has positive leading coefficient.