

Question 9

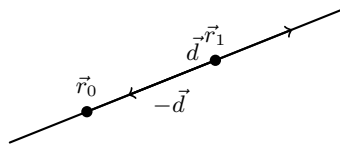
Show that

$$L_1 : \vec{r} = \langle 2, -1, 3 \rangle + t \langle 4, 2, -2 \rangle \quad \text{and} \quad L_2 : \vec{r} = \langle 8, 2, 0 \rangle + s \langle -2, -1, 1 \rangle$$

describe the same geometric line. Find the parameter conversion between t and s .

Question 9 – Solution

Strategy Check parallel directions and show that one base point lies on the other line.



different parameters, same point set

Same point set The second direction is $-\frac{1}{2}\langle 4, 2, -2 \rangle$. Also $\langle 8, 2, 0 \rangle = \langle 2, -1, 3 \rangle + \frac{3}{2}\langle 4, 2, -2 \rangle$, so the second base point lies on L_1 . Thus the lines coincide.

Parameter conversion Equating the representations gives

$$\vec{r} = \vec{r}_0 + t\vec{d} = \vec{r}_0 + \frac{3}{2}\vec{d} - \frac{s}{2}\vec{d},$$

so $\boxed{t = \frac{3}{2} - \frac{s}{2}}$, equivalently $s = 3 - 2t$.

Verification The conversion makes all three coordinates identical.