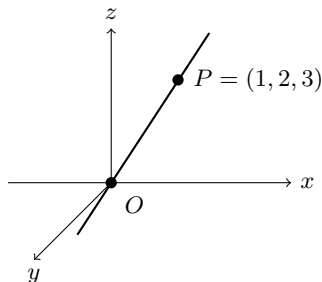


Question 8

Find the line that passes through $P = (1, 2, 3)$ and meets both the x -axis and the y -axis. Determine the two intercept points, or prove that no such line exists.

Question 8 – Solution

Strategy Distinct axis intercepts determine a line in the xy -plane, but both intercepts may be the origin.



Distinct intercepts A line through distinct points $X = (a, 0, 0)$ and $Y = (0, b, 0)$ lies in $z = 0$, so it cannot contain P .

Coincident intercepts The axes share the origin. The line through the origin and P is

$$\vec{r}(t) = t \langle 1, 2, 3 \rangle, \quad t \in \mathbb{R}.$$

It passes through P at $t = 1$ and meets each axis at $t = 0$. Thus both intercepts are $(0, 0, 0)$.

Key idea The distinct-point argument does not rule out coincident intercepts. This is the unique line satisfying the question.