

Question 6

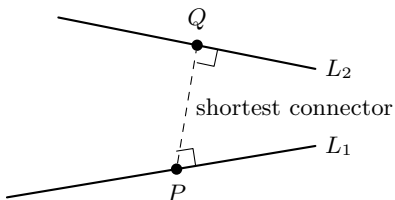
Find the shortest distance between the skew lines

$$L_1 : \vec{r} = \langle 0, 0, 0 \rangle + s \langle 1, 1, 0 \rangle, \quad L_2 : \vec{r} = \langle 0, 1, 1 \rangle + t \langle 1, -1, 0 \rangle,$$

and identify the pair of closest points.

Question 6 – Solution

Strategy The connector between closest points must be perpendicular to both line directions.



Closest points Let $P = (s, s, 0)$ and $Q = (t, 1 - t, 1)$. Requiring $(Q - P) \cdot \langle 1, 1, 0 \rangle = 0$ gives $1 - 2s = 0$, so $s = 1/2$. Requiring $(Q - P) \cdot \langle 1, -1, 0 \rangle = 0$ gives $2t - 1 = 0$, so $t = 1/2$.

Distance Thus $P = (1/2, 1/2, 0)$ and $Q = (1/2, 1/2, 1)$, with

$$\boxed{d(L_1, L_2) = |PQ| = 1}.$$

Verification $Q - P = \langle 0, 0, 1 \rangle$ is perpendicular to both direction vectors.