

Question 5

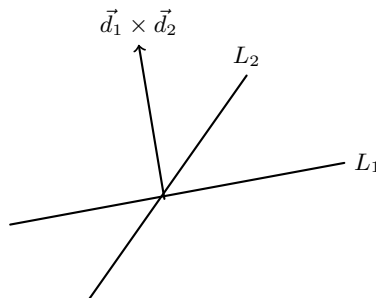
Find the line through $P = (2, -1, 4)$ whose direction is perpendicular to the directions of both lines

$$L_1 : \vec{r} = \langle 0, 1, 2 \rangle + s \langle 1, 2, -1 \rangle, \quad L_2 : \vec{r} = \langle 3, 0, -1 \rangle + t \langle 2, -1, 1 \rangle.$$

Explain why exactly one such direction exists.

Question 5 – Solution

Strategy A vector perpendicular to both directions is their cross product.



Direction Compute

$$\langle 1, 2, -1 \rangle \times \langle 2, -1, 1 \rangle = \langle 1, -3, -5 \rangle.$$

Therefore the requested line is

$$\boxed{\vec{r} = \langle 2, -1, 4 \rangle + u \langle 1, -3, -5 \rangle}.$$

Verification The new direction has dot product 0 with each original direction.

Uniqueness The original directions are not parallel, so their common perpendicular directions form the one-dimensional span of their nonzero cross product; reversing direction describes the same line.