

Question 4

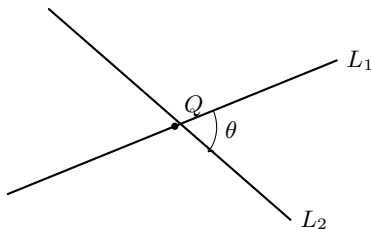
The lines

$$L_1 : \vec{r} = \langle 1, 2, -1 \rangle + s \langle 2, -1, 2 \rangle, \quad L_2 : \vec{r} = \langle 5, 0, 3 \rangle + t \langle 1, 2, -2 \rangle$$

intersect. Find their intersection and the acute angle between them.

Question 4 – Solution

Strategy Solve the coordinate equations for the intersection; use direction vectors for the angle.



Intersection Equating coordinates gives $1 + 2s = 5 + t$, $2 - s = 2t$, and $-1 + 2s = 3 - 2t$. The solution is $s = 2, t = 0$, so $Q = (5, 0, 3)$.

Angle With $\vec{d}_1 = \langle 2, -1, 2 \rangle$ and $\vec{d}_2 = \langle 1, 2, -2 \rangle$,

$$|\vec{d}_1 \cdot \vec{d}_2| = 4, \quad \|\vec{d}_1\| = 3, \quad \|\vec{d}_2\| = 3.$$

Hence $\theta = \cos^{-1}(4/9) \approx 63.6^\circ$.

Verification Substituting $s = 2$ and $t = 0$ gives the same point on both lines.