

Question 2

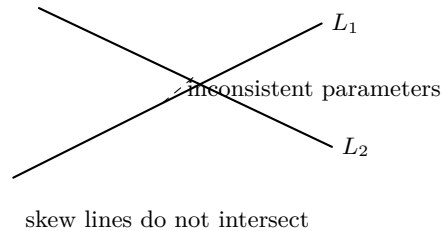
Determine whether the lines

$$L_1 : \vec{r} = \langle 1, 0, 2 \rangle + s \langle 2, -1, 3 \rangle, \quad L_2 : \vec{r} = \langle 3, -1, 1 \rangle + t \langle 1, 1, 2 \rangle$$

intersect, are parallel, or are skew. Justify the classification.

Question 2 – Solution

Strategy Equate coordinates. An intersection requires one pair (s, t) satisfying all three equations.



Coordinate equations The first two equations are $1 + 2s = 3 + t$ and $-s = -1 + t$. The second gives $t = 1 - s$; substitution into the first gives $s = 1$, hence $t = 0$.

Consistency check The third coordinates would be $2 + 3(1) = 5$ and $1 + 2(0) = 1$, so they disagree. The directions are not scalar multiples. Therefore the lines are skew.

Key idea Solving only two coordinate equations can produce a false intersection; the remaining coordinate is essential.