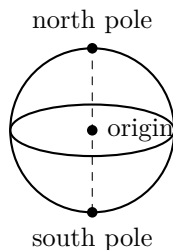


Question 10

Under the standard ranges $\rho \geq 0$, $0 \leq \theta < 2\pi$, $0 \leq \phi \leq \pi$, discuss exactly where spherical coordinates fail to be unique and why.

Question 10 – Solution

Strategy Examine the conversion formulas at the origin and on the polar axis.



Origin If $\rho = 0$, then $x = y = z = 0$ for every θ and ϕ . Thus both angles are arbitrary at the origin.

Polar axis At $\phi = 0$ or $\phi = \pi$, $\sin \phi = 0$, so $x = y = 0$ regardless of θ . Therefore azimuth is arbitrary at the north and south polar rays.

Elsewhere For $\rho > 0$ and $0 < \phi < \pi$, the height uniquely determines $\phi = \arccos(z/\rho)$, and the nonzero projection (x, y) uniquely determines θ in $[0, 2\pi)$. Thus coordinates are unique away from the origin and z -axis.