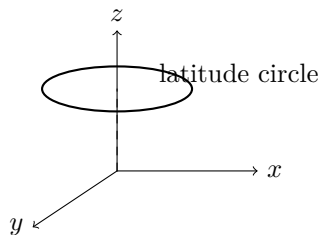


Question 9

A curve on the sphere $\rho = R$ has constant polar angle $\phi = \alpha$, while $\theta = t$ varies from 0 to 2π . Find a rectangular parameterization and the exact length of the curve.

Question 9 – Solution

Strategy Convert the fixed spherical coordinates and recognize a horizontal circle.



Parameterization

$$\vec{r}(t) = \langle R \sin \alpha \cos t, R \sin \alpha \sin t, R \cos \alpha \rangle, \quad 0 \leq t \leq 2\pi.$$

Length Differentiation gives

$$\|\vec{r}'(t)\| = R \sin \alpha$$

for $0 \leq \alpha \leq \pi$. Hence

$$L = 2\pi R \sin \alpha.$$

Geometry The latitude circle has Euclidean radius $R \sin \alpha$ and lies in the horizontal plane $z = R \cos \alpha$. At either pole, this radius and length are zero.