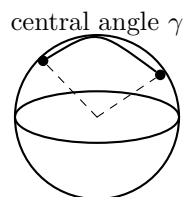


Question 8

Two points have spherical coordinates $(\rho_1, \theta_1, \phi_1)$ and $(\rho_2, \theta_2, \phi_2)$. Derive a formula for the square of their distance using their central angle.

Question 8 – Solution

Strategy Write the spherical unit direction vectors and use the dot-product form of the law of cosines.



Direction dot product If γ is the angle between the radial directions, then

$$\cos \gamma = \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 \cos(\theta_1 - \theta_2).$$

Distance With position vectors $\vec{p}_1, \vec{p}_2$,

$$d^2 = \|\vec{p}_1 - \vec{p}_2\|^2 = \rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos \gamma.$$

Therefore

$$d^2 = \rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 [\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 \cos(\theta_1 - \theta_2)].$$