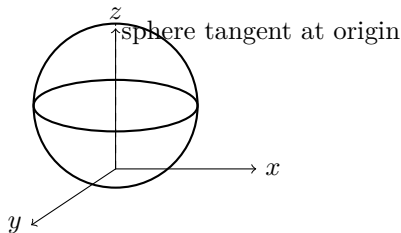


Question 3

Convert the spherical equation $\rho = 4 \cos \phi$ to rectangular coordinates and identify the surface, including its center and radius. Explain why the origin belongs to it.

Question 3 – Solution

Strategy Multiply by ρ and use $\rho^2 = x^2 + y^2 + z^2$ and $\rho \cos \phi = z$.



Conversion

$$\rho^2 = 4\rho \cos \phi \implies x^2 + y^2 + z^2 = 4z.$$

Completing the square,

$$\boxed{x^2 + y^2 + (z - 2)^2 = 4}.$$

Geometry The surface is a sphere of radius 2 centered at $(0, 0, 2)$. It contains the origin because the distance from the origin to the center is exactly the radius. In the original spherical equation, $\rho = 0$ and $\phi = \pi/2$ represent the origin and satisfy $0 = 4 \cos(\pi/2)$.