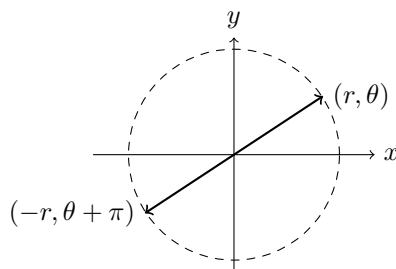


Question 10

Explain why cylindrical coordinates are not unique if negative values of r are allowed. Prove the identity relating (r, θ, z) and $(-r, \theta + \pi, z)$, then state a standard uniqueness convention and its remaining exception.

Question 10 – Solution

Strategy Compare the rectangular coordinates produced by the two triples.



Identity

$$(-r) \cos(\theta + \pi) = (-r)(-\cos \theta) = r \cos \theta,$$

$$(-r) \sin(\theta + \pi) = (-r)(-\sin \theta) = r \sin \theta.$$

The z -coordinate is unchanged, so

$$\boxed{(r, \theta, z) \sim (-r, \theta + \pi, z)}.$$

Convention Requiring $r \geq 0$ and $0 \leq \theta < 2\pi$ makes coordinates unique whenever $r > 0$. On the z -axis, $r = 0$ and every angle represents the same point, so θ remains intrinsically undetermined there.