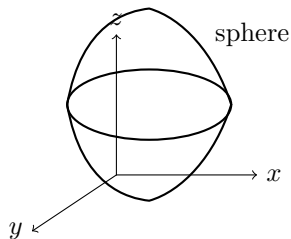


Question 5

Convert $x^2 + y^2 + z^2 = 6z$ to cylindrical coordinates and identify the surface. Describe the ranges of r and z on it.

Question 5 – Solution

Strategy Complete the square in z after replacing $x^2 + y^2$ by r^2 .



Equation

$$r^2 + z^2 = 6z \iff \boxed{r^2 + (z - 3)^2 = 9}.$$

Geometry This is a sphere of radius 3 centered at $(0, 0, 3)$. It is independent of θ , reflecting rotational symmetry about the z -axis.

Ranges Since $r^2 \leq 9$, $\boxed{0 \leq r \leq 3}$. The vertical extent is $\boxed{0 \leq z \leq 6}$. At a fixed height z , the horizontal trace has radius $\sqrt{9 - (z - 3)^2}$.