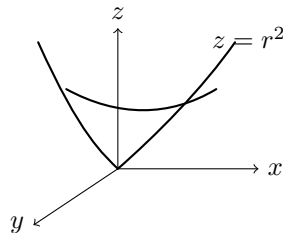


Question 3

Convert the paraboloid $z = x^2 + y^2$ to cylindrical coordinates. Describe its traces for fixed z and for fixed θ , and explain why the new equation reveals its symmetry.

Question 3 – Solution

Strategy Use $x^2 + y^2 = r^2$ and interpret one coordinate at a time.



Equation

$$\boxed{z = r^2}, \quad r \geq 0.$$

Horizontal traces For $z = c \geq 0$, $r = \sqrt{c}$, a circle centered on the z -axis. There are no points for $c < 0$.

Vertical traces Fixing θ selects a vertical half-plane through the z -axis; within it, $z = r^2$ is one half of an upward-opening parabola (including its vertex).

Symmetry The equation contains no θ , so rotating any point about the z -axis preserves the surface.