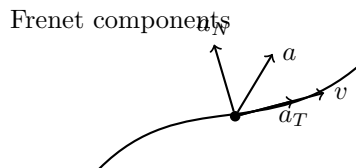


Question 10

For a regular motion with speed $v = ds/dt$, prove the Frenet acceleration formula $\vec{a} = (dv/dt)\mathbf{T} + \kappa v^2\mathbf{N}$. Identify the geometric role of each term.

Question 10 – Solution

Strategy Write velocity as speed times unit tangent and apply the product and chain rules.



Derivation

$$\vec{v} = v\mathbf{T}.$$

Therefore

$$\vec{a} = \frac{d}{dt}(v\mathbf{T}) = \frac{dv}{dt}\mathbf{T} + v\frac{d\mathbf{T}}{ds}\frac{ds}{dt}.$$

Since $d\mathbf{T}/ds = \kappa\mathbf{N}$ and $ds/dt = v$,

$$\boxed{\vec{a} = \frac{dv}{dt}\mathbf{T} + \kappa v^2\mathbf{N}}.$$

Meaning The tangential term changes speed. The normal term changes direction and points toward the local center of curvature; its magnitude grows quadratically with speed.