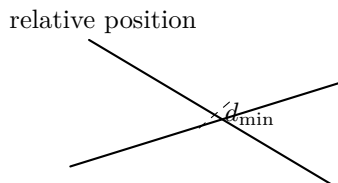


Question 9

Two particles move according to $\vec{r}_1(t) = \langle t, 2t, 0 \rangle$ and $\vec{r}_2(t) = \langle 4 - t, 1, 2t \rangle$ for $t \geq 0$. Find the time and value of their minimum separation.

Question 9 – Solution

Strategy Minimize squared distance; it has the same minimizer as distance and avoids a square root.



Relative position

$$\vec{d}(t) = \vec{r}_1 - \vec{r}_2 = \langle 2t - 4, 2t - 1, -2t \rangle.$$

Thus

$$D^2(t) = (2t - 4)^2 + (2t - 1)^2 + (-2t)^2 = 12t^2 - 20t + 17.$$

Minimum The vertex occurs at $t = 20/24 = 5/6$. Then

$$D^2\left(\frac{5}{6}\right) = \frac{26}{3}, \quad \boxed{D_{\min} = \sqrt{\frac{26}{3}}}.$$

Because the quadratic opens upward and $5/6 \geq 0$, this is the required global minimum.