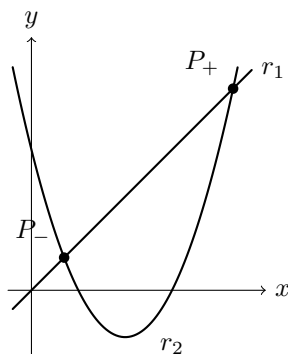


Question 8

Particles follow $\vec{r}_1(t) = \langle t, t, 0 \rangle$ and $\vec{r}_2(t) = \langle 2 - t, t^2 - 1, 0 \rangle$. Do their paths intersect? Do the particles collide for $t \geq 0$? Clearly distinguish the two questions.

Question 8 – Solution

Strategy Path intersection may use different parameter values; collision requires equal positions at the same time.



Path intersection Write the first path as $y = x$. On the second, $x = 2 - u$ and $y = u^2 - 1$. Thus

$$u^2 + u - 3 = 0, \quad u = \frac{-1 \pm \sqrt{13}}{2}.$$

The full paths intersect at

$$\left(\frac{5 \mp \sqrt{13}}{2}, \frac{5 \mp \sqrt{13}}{2}, 0 \right).$$

If both paths are restricted to nonnegative times, only the point with $5 - \sqrt{13}$ remains.

Collision test At the same time t , equality of x -coordinates requires $t = 2 - t$, so $t = 1$. But

$$\vec{r}_1(1) = \langle 1, 1, 0 \rangle, \quad \vec{r}_2(1) = \langle 1, 0, 0 \rangle.$$

Therefore the particles do not collide. The geometric intersections occur at different parameter times.