

Question 6

Prove that a particle with nonzero velocity is speeding up exactly when $\vec{v} \cdot \vec{a} > 0$, and slowing down exactly when $\vec{v} \cdot \vec{a} < 0$.

Question 6 – Solution

Strategy Differentiate speed through its square to avoid differentiating a square root prematurely.

$$v \cdot a > 0$$



Derivation Let $v = \|\vec{v}\| > 0$. Then

$$v^2 = \vec{v} \cdot \vec{v}.$$

Differentiating gives $2v \, dv/dt = 2\vec{v} \cdot \vec{a}$, so

$$\boxed{\frac{dv}{dt} = \frac{\vec{v} \cdot \vec{a}}{\|\vec{v}\|}}.$$

Conclusion The denominator is positive. Therefore the sign of the speed derivative is exactly the sign of $\vec{v} \cdot \vec{a}$: positive means speeding up, negative means slowing down, and zero means instantaneously constant speed.