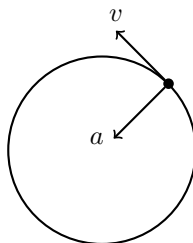


Question 3

A particle moves on the circle $\vec{r}(t) = \langle R \cos(\omega t), R \sin(\omega t) \rangle$, where $R, \omega > 0$. Find velocity, acceleration, speed, and show that acceleration points toward the center with magnitude v^2/R .

Question 3 – Solution

Strategy Differentiate twice and compare acceleration with the position vector.



Derivatives

$$\vec{v} = R\omega \langle -\sin \omega t, \cos \omega t \rangle, \quad \vec{a} = -R\omega^2 \langle \cos \omega t, \sin \omega t \rangle = -\omega^2 \vec{r}.$$

Thus $\boxed{v = R\omega}$ and acceleration is radially inward.

Magnitude

$$\|\vec{a}\| = R\omega^2 = \frac{(R\omega)^2}{R} = \boxed{\frac{v^2}{R}}.$$

Velocity is tangent to the circle and perpendicular to the centripetal acceleration.