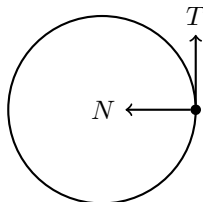


Question 10

The circle of radius 2 is parameterized nonuniformly by $\vec{r}(t) = \langle 2 \cos(t^3), 2 \sin(t^3) \rangle$ for $t > 0$. Compute its curvature directly and explain why the changing speed does not affect the result.

Question 10 – Solution

Strategy Let $\theta = t^3$ and use the turning-angle relation $\kappa = \|d\mathbf{T}/dt\|/(ds/dt)$.



nonuniform parameter, same curvature

Tangent and rates Since $\theta' = 3t^2 > 0$,

$$\mathbf{T} = \langle -\sin \theta, \cos \theta \rangle, \quad \left\| \frac{d\mathbf{T}}{dt} \right\| = 3t^2.$$

The speed is $ds/dt = \|\vec{r}'\| = 2(3t^2) = 6t^2$.

Curvature

$$\kappa = \frac{\|d\mathbf{T}/dt\|}{ds/dt} = \frac{3t^2}{6t^2} = \frac{1}{2}.$$

Interpretation Curvature measures direction change per unit distance, not per unit time. Both direction-change rate and speed acquire the same parameter factor, which cancels.