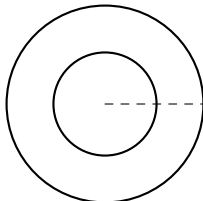


**Question 9**

A regular curve  $\vec{r}(t)$  is enlarged by a factor  $c > 0$  to form  $\vec{q}(t) = c\vec{r}(t)$ . Prove that  $\kappa_q = \kappa_r/c$ . What happens to the radius of curvature?

## Question 9 – Solution

**Strategy** Track how first derivatives, second derivatives, cross products, and norms scale.



dilation by  $c$  lowers curvature by  $1/c$

**Scaling**

$$\vec{q}' = c\vec{r}', \quad \vec{q}'' = c\vec{r}'', \quad \|\vec{q}' \times \vec{q}''\| = c^2 \|\vec{r}' \times \vec{r}''\|.$$

Also  $\|\vec{q}'\|^3 = c^3 \|\vec{r}'\|^3$ . Hence

$$\kappa_q = \frac{c^2}{c^3} \kappa_r = \frac{\kappa_r}{c}.$$

**Radius** Since  $\rho = 1/\kappa$ ,  $\boxed{\rho_q = c\rho_r}$ . Enlargement makes every local osculating circle  $c$  times larger and the curve correspondingly less sharply bent.