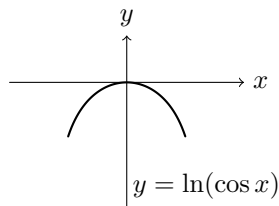


Question 8

On $-\frac{\pi}{2} < x < \frac{\pi}{2}$, determine the curvature of $y = \ln(\cos x)$. Explain why the stated domain makes the simplification unambiguous.

Question 8 – Solution

Strategy Differentiate twice and reduce the denominator using $1 + \tan^2 x = \sec^2 x$.



Calculation

$$y' = -\tan x, \quad y'' = -\sec^2 x.$$

Therefore

$$\kappa(x) = \frac{\sec^2 x}{(1 + \tan^2 x)^{3/2}} = \frac{\sec^2 x}{|\sec x|^3}.$$

On the stated interval, $\cos x > 0$ and $\sec x > 0$, so

$$\boxed{\kappa(x) = \cos x}.$$

Behavior The maximum curvature is 1 at $x = 0$, while $\kappa \rightarrow 0$ as the graph approaches either vertical asymptote.