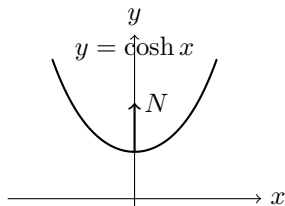


Question 7

For the catenary $y = \cosh x$, find $\kappa(x)$ and the radius of curvature $\rho(x)$. Where is the curve most sharply bent, and what happens as $|x| \rightarrow \infty$?

Question 7 – Solution

Strategy Use the graph formula $\kappa = |y''|/(1 + y'^2)^{3/2}$ and a hyperbolic identity.



Calculation Since $y' = \sinh x$, $y'' = \cosh x$, and $1 + \sinh^2 x = \cosh^2 x$,

$$\kappa(x) = \frac{\cosh x}{\cosh^3 x} = \operatorname{sech}^2 x, \quad \rho(x) = \cosh^2 x.$$

Analysis Curvature is largest at $x = 0$, where $\kappa = 1$. As $|x| \rightarrow \infty$, $\kappa \rightarrow 0$ and $\rho \rightarrow \infty$, so the arms become locally less sharply bent.