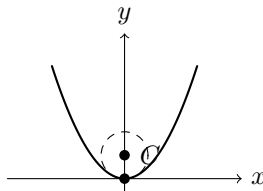


Question 6

Find the osculating circle of the parabola $y = x^2$ at its vertex. State its center, radius, and equation, and explain which side of the tangent contains the center.

Question 6 – Solution

Strategy Use curvature for the radius and the principal normal for the center direction.



Curvature and normal From Question 2, $\kappa(0) = 2$, so $\rho = 1/\kappa = 1/2$. At the vertex, the unit tangent points right and the parabola bends upward, so $\mathbf{N} = \langle 0, 1 \rangle$.

Center and equation

$$C = (0, 0) + \frac{1}{2} \langle 0, 1 \rangle = \left(0, \frac{1}{2} \right),$$

$$\boxed{x^2 + \left(y - \frac{1}{2} \right)^2 = \frac{1}{4}}.$$

Geometry The center lies on the concave side of the curve, above the horizontal tangent. The circle has second-order contact with the parabola at the vertex.