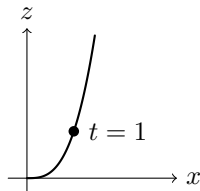


Question 4

Compute the curvature of the twisted cubic $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at $t = 1$. Show the cross product and simplify the exact value.

Question 4 – Solution

Strategy Use the parameter-independent cross-product formula.



Derivatives

$$\vec{r}' = \langle 1, 2t, 3t^2 \rangle, \quad \vec{r}'' = \langle 0, 2, 6t \rangle,$$

$$\vec{r}' \times \vec{r}'' = \langle 6t^2, -6t, 2 \rangle.$$

At $t = 1$, its magnitude is $2\sqrt{19}$, while $\|\vec{r}'(1)\| = \sqrt{14}$. Therefore

$$\kappa(1) = \frac{2\sqrt{19}}{(\sqrt{14})^3} = \frac{\sqrt{19}}{7\sqrt{14}}.$$

Check The cross product is nonzero, so the tangent is genuinely changing direction and the curvature is positive.