

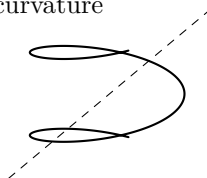
Question 3

For the helix $\vec{r}(t) = \langle 3 \cos t, 3 \sin t, 4t \rangle$, compute the curvature for all t . Explain geometrically why a constant answer is reasonable.

Question 3 – Solution

Strategy Compute speed and the magnitude of $\vec{r}' \times \vec{r}''$ separately.

constant curvature



Calculation

$$\vec{r}' = \langle -3 \sin t, 3 \cos t, 4 \rangle, \quad \vec{r}'' = \langle -3 \cos t, -3 \sin t, 0 \rangle,$$

$$\vec{r}' \times \vec{r}'' = \langle 12 \sin t, -12 \cos t, 9 \rangle.$$

Thus $\|\vec{r}'\| = 5$ and $\|\vec{r}' \times \vec{r}''\| = 15$, giving

$$\kappa = \frac{15}{5^3} = \frac{3}{25}.$$

Geometry Every point of a circular helix has the same local shape: rotation about and translation along its axis carry one point to another, so its bending is uniform.