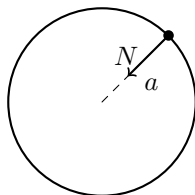


Question 1

Starting from $\vec{r}(t) = \langle a \cos t, a \sin t \rangle$, where $a > 0$, derive the curvature of a circle. Interpret the result in terms of turning sharpness and radius of curvature.

Question 1 – Solution

Strategy Use $\kappa = \|\vec{r}' \times \vec{r}''\| / \|\vec{r}'\|^3$, viewing planar vectors as three-dimensional vectors with zero third component.



Calculation $\vec{r}' = \langle -a \sin t, a \cos t, 0 \rangle$ and $\vec{r}'' = \langle -a \cos t, -a \sin t, 0 \rangle$. Thus $\|\vec{r}'\| = a$ and $\|\vec{r}' \times \vec{r}''\| = a^2$. Therefore

$$\boxed{\kappa = \frac{a^2}{a^3} = \frac{1}{a}}, \quad \boxed{\rho = \frac{1}{\kappa} = a}.$$

Interpretation Small circles turn more sharply and have larger curvature; the radius of curvature ρ is exactly the circle's geometric radius.