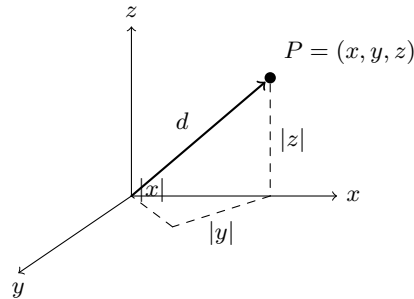


Question 10

Derive the distance formula between $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ using the Pythagorean theorem twice. Then use it to find all values of t for which the distance from $(t, 2t, -1)$ to $(1, -2, 3)$ is 6.

Question 10 – Solution

Strategy First combine the x - and y -changes into a planar diagonal; then combine that diagonal with the z -change.



Derivation The planar displacement has length $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. A second right triangle therefore gives

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Application Set the squared distance equal to 36:

$$(t - 1)^2 + (2t + 2)^2 + (-4)^2 = 36.$$

This becomes $5t^2 + 6t - 15 = 0$, so

$$t = \frac{-3 \pm 2\sqrt{21}}{5}.$$

Verification Substitution into the quadratic recovers squared distance 36; both values are valid because distance is nonnegative.