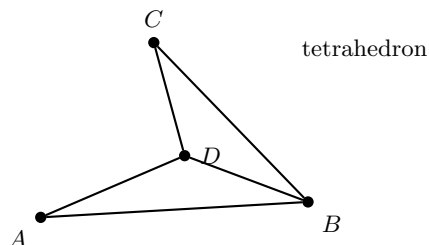


Question 9

Four points are $A = (1, 1, 1)$, $B = (5, 1, 1)$, $C = (3, 1 + 2\sqrt{3}, 1)$, and $D = (3, 1 + 2/\sqrt{3}, 1 + 4\sqrt{2/3})$. Prove that they are the vertices of a regular tetrahedron and find its center.

Question 9 – Solution

Strategy A tetrahedron is regular if all six pairwise distances agree. Symmetry suggests that its center is the average of its four vertices.



Edge lengths Computation gives $AB = AC = BC = 4$. Also,

$$AD^2 = BD^2 = 4 + \frac{4}{3} + \frac{32}{3} = 16, \quad CD^2 = \frac{16}{3} + \frac{32}{3} = 16.$$

Thus all six edges have length 4, proving the tetrahedron is regular.

Center Averaging the vertices gives

$$G = \left(3, 1 + \frac{2\sqrt{3}}{3}, 1 + \sqrt{\frac{2}{3}} \right).$$

Verification By symmetry—or by direct substitution— G is equidistant from all four vertices.