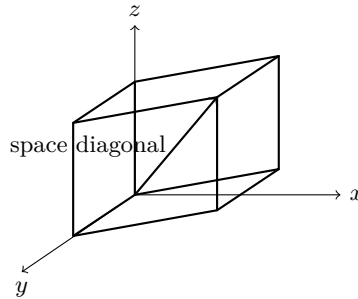


Question 8

A rectangular box has one vertex at the origin and the opposite vertex $P = (a, b, c)$ in the first octant. Its space diagonal has length 13, its xy -face diagonal has length 5, and the y -edge is longer than the x -edge. If the side lengths are positive integers, determine P .

Question 8 – Solution

Strategy Use the face diagonal to determine two integer side lengths, then use the space diagonal for the third.



Base dimensions The positive integer side lengths satisfying $a^2 + b^2 = 25$ are 3 and 4. Since the y -edge is longer than the x -edge, $a = 3$ and $b = 4$.

Third dimension The space diagonal gives

$$a^2 + b^2 + c^2 = 169 \implies 25 + c^2 = 169,$$

so $c = 12$. Therefore $\boxed{P = (3, 4, 12)}$.

Verification The face diagonal is $\sqrt{3^2 + 4^2} = 5$ and the space diagonal is $\sqrt{3^2 + 4^2 + 12^2} = 13$.