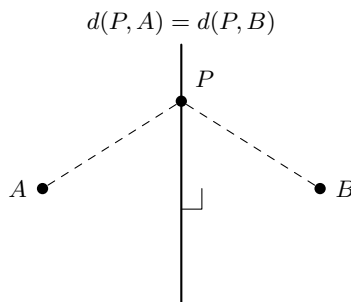


Question 6

Describe geometrically the set of all points $P = (x, y, z)$ equidistant from $A = (2, -1, 3)$ and $B = (-4, 5, 1)$. Derive a Cartesian equation for the set and show that the midpoint of AB belongs to it.

Question 6 – Solution

Strategy Equate squared distances. Quadratic terms cancel, revealing the perpendicular-bisector surface.



Derivation Set

$$(x - 2)^2 + (y + 1)^2 + (z - 3)^2 = (x + 4)^2 + (y - 5)^2 + (z - 1)^2.$$

After expanding and simplifying, $\boxed{3x - 3y + z + 7 = 0}$.

Interpretation This is the plane perpendicular to $\overrightarrow{AB} = \langle -6, 6, -2 \rangle$ through the midpoint $M = (-1, 2, 2)$. Indeed, $3(-1) - 3(2) + 2 + 7 = 0$.

Verification Interchanging A and B does not change the locus, and every point in the plane has equal squared distance to the endpoints.