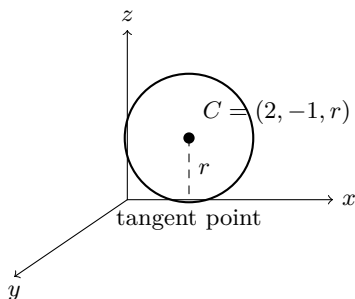


Question 5

A sphere has center $C = (2, -1, c)$, is tangent to the xy -plane, lies entirely above that plane, and passes through $P = (5, 3, 2)$. Find c and the sphere's equation.

Question 5 – Solution

Strategy Tangency to the xy -plane makes the radius equal to the center's height. The condition that the sphere lies above the plane forces $c > 0$.



Build the equation Here $r = c$. Because P is on the sphere,

$$(5 - 2)^2 + (3 + 1)^2 + (2 - c)^2 = c^2.$$

Thus $25 + (2 - c)^2 = c^2$, so $29 - 4c = 0$ and $c = 29/4$.

Result The radius is $29/4$, and

$$\boxed{(x - 2)^2 + (y + 1)^2 + \left(z - \frac{29}{4}\right)^2 = \left(\frac{29}{4}\right)^2}.$$

Verification The lowest z -coordinate is $c - r = 0$, so the sphere is tangent to the plane and otherwise lies above it.