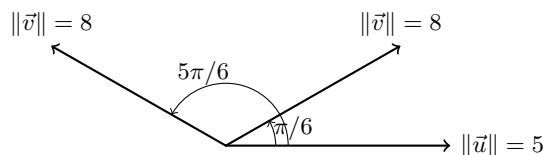


Question 10

Vectors $\vec{u}$ and $\vec{v}$ have lengths 5 and 8, and $\|\vec{u} \times \vec{v}\| = 20$. Find every possible angle between them in $[0, \pi]$.

Question 10 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Use $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\|\sin\theta$: $20 = 40\sin\theta$.

Step 2 Thus $\sin\theta = 1/2$.

Step 3 On $[0, \pi]$, the two solutions are $\theta = \pi/6$ or $5\pi/6$. The cross-product magnitude cannot distinguish acute from obtuse orientation.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.