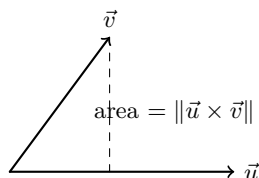


Question 8

Prove the area identity $\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$ using the angle between the vectors.

Question 8 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 If either vector is zero, both sides are zero. Otherwise, let θ be the angle between $\vec{u}$ and $\vec{v}$. Then $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\|\sin\theta$.

Step 2 Squaring and using $\sin^2\theta = 1 - \cos^2\theta$ gives $\|\vec{u}\|^2\|\vec{v}\|^2(1 - \cos^2\theta)$.

Step 3 Since $\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta$, substitution yields the claimed identity. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.