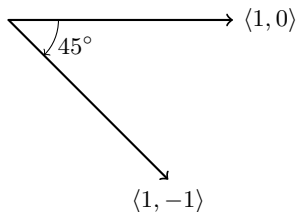


Question 9

For which real t is the angle between $\langle 1, t \rangle$ and $\langle 1, -1 \rangle$ equal to 45° ?

Question 9 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Use the cosine formula: $(1 - t)/(\sqrt{1 + t^2}\sqrt{2}) = \sqrt{2}/2$.

Step 2 This simplifies to $1 - t = \sqrt{1 + t^2}$. The right side is nonnegative, so $t \leq 1$. Squaring gives $1 - 2t + t^2 = 1 + t^2$, hence $t = 0$.

Step 3 It satisfies the original equation, so $\boxed{t = 0}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.