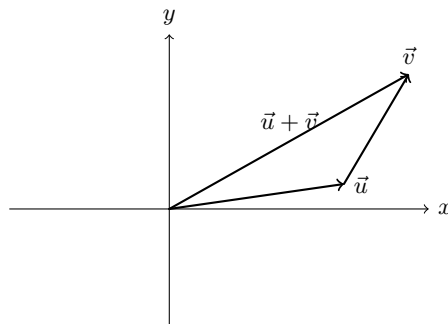


Question 8

Prove the identity $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2$.

Question 8 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Start with the squared norm: $\|\vec{u} + \vec{v}\|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$.

Step 2 Distribute the dot product to obtain $\vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$.

Step 3 Use symmetry and $\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$ to get the stated identity. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.